B–12–X

Roll No. .....

Total No. of Questions : 29]

[Total No. of Printed Pages : 8

XIIARJKUT23
9112–X
MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 100

SECTION–A

(MULTIPLE CHOICE QUESTIONS)

1 each

1. Range of the function $f : R \to R$ defined as $f(x) = x^2$ is :

(A) $(0, \infty)$

(B) $(-\infty, 0)$

(C) $[0, \infty)$

(D) $(-\infty, 0]$

2. $\sin^{-1} x + \cos^{-1} x$, $x \in [-1, 1]$ is equal to :

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) π

3. If A is a square matrix of order n, then A(adj A) = ...

(A) $|A|$

(B) I

(C) $|A| I$

(D) None of these

. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then $\vec{a} \cdot \vec{b}$ = ...

(A) $\cos \theta$

(B) $\sin \theta$

(C) $ab \cos \theta$

(D) $ab \sin \theta$

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

5. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, find the values of x.

6. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

7. Differentiate $\sin(x^2 + 5)$ with respect to x.

8. Find :

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

9. A coin is tossed three times. Find P(E/F), where E is the event "head on third toss" and F is the event "heads on first two tosses".

10. Compute P(A ∩ B), where P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4.

11. Find the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ and having magnitude of 8 units.

12. Define Linear Constraints.

SECTION–C

(SHORT ANSWER TYPE QUESTIONS) 4 each

13. Find gof and fog if $f(x) = |x|$ and $g(x) = |5x - 2|$.

14. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, find the value of x.

15. If :

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

prove that $A^3 - 6A^2 + 7A + 2I = 0$.

16. Find local maxima and local minima if any of the function :

$$f(x) = x^3 - 6x^2 + 9x + 15$$

17. Find general solution of differential equation :

$$x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$$

18. Find $\frac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$.

19. Prove that :

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$$

20. If $y = (\tan^{-1} x)^2$, show that :

$$(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$$

21. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

22. Find the Vector and Cartesian equations of the line that passes through the points (3, –2, –5) and (3, –2, 6).

23. Solve the following graphically :

Minimise :

$$Z = x + 2y$$

Subject to the constraints :

$$2x + y \geq 3$$

$$x + 2y \geq 6$$

$$x, y \geq 0$$

SECTION–D

(LONG ANSWER TYPE QUESTIONS) 6 each

24. Using properties of determinants show that :

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a - b)\ (b - c)\ (c - a)\ (ab + bc + ca)$$

Or

Solve the system of linear equations using matrix method :

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

25. Find $\frac{dy}{dx}$ if $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$.

Or

Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$.

26. Find $\int_0^{\pi/2} \sqrt{\sin\phi}\cos^5\phi\, d\phi$.

Or

Using integration, find the area of region bounded by triangle whose vertices are (–1, 0), (1, 3), (3, 2).

27. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d} = 15$.

Or

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

28. Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or decreasing.

Or

Evaluate :

$$\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\, dx$$

29. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item.

Or

Find the mean and variance of the number obtained on a throw of an unbiased die.

A–6–A

Roll No. .....

Total No. of Questions : **29**]

[Total No. of Printed Pages : **8**

12thSZARJD22

6006–A

MATHEMATICS

Time : 2.30 Hours]

[Maximum Marks : 100

Section–A

(Multiple Choice Questions) 1 each

1. The relation R in the set {1, 2, 3} given by R = {(x, y) | $x < y$, $x, y \in A$} is :

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) Anti-symmetric

2. The principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is :

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{3\pi}{4}$

3. Two matrices A and B of the same order are said to be equal, if :

(A) $a_{ij} = 0$ (B) $b_{ij} = 0$

(C) $a_{ij} + b_{ij} = 0$ (D) $a_{ij} = b_{ij}$ for all i, j

4. The direction cosines of a unit vector along x-axis are :

(A) (1, 0, 0) (B) (0, 1, 0)

(C) (0, 0, 1) (D) (1, 1, 1)

Section–B

(Very Short Answer Type Questions) 2 each

5. Find the values of x, y and z from the following equation :

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

6. Evaluate :

$$\int (ax^2 + bx + c)dx$$

7. Find the rate of change of area of a circle with respect to its radius when $r = 5$ cm.

8. Form the differential equation representing the family of curve $y = mx^2$, where m is arbitrary constant.

9. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

10. Solve the following L.P.P. graphically :

Maximise :

$$Z = 3x + 4y$$

Subject to constraints :

$$x + y \leq 4,$$

$$x \geq 0, \; y \geq 0$$

11. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

12. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

Section–C

(Short Answer Type Questions) 4 each

13. Show that $f : (-1\ \ 1) \to R$ given by $f(x) = \dfrac{x}{x+2}$ is one-one. Find the inverse of the function $f : (-1\ \ 1) \to$ Range of f.

14. Solve :

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

15. For the matrix A and B verify that $(AB)' = B'A'$, $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$.

16. Find the relationship between a and b so that the function f defined by :

$$f(x) = \begin{cases} ax + 1 & \text{if} \quad x \le 3 \\ bx + 3 & \text{if} \quad x > 3 \end{cases}$$

is continuous at $x = 3$.

17. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is :

(a) Strictly increasing

(b) Strictly decreasing

18. Find the equations of the tangent and normal to the given curve at the indicated point $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5).

19. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

20. Find the area of the region bounded by the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

21. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

22. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

23. Solve the following problem graphically :

Minimise and maximise : $Z = 3x + 9y$

Subject to the linear constraints :

$$x + 3y \leq 60,\ x \leq y,$$

and

$$x + y \geq 10,\ x \geq 0,\ y \geq 0$$

Section–D

(Long Answer Type Questions)

24. By using properties of determinants prove that :

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

Or

If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that A(adj A) = | A |I. Also find A^{-1}.

25. Find $\frac{dy}{dx}$ of the function $x^y + y^x = 1$.

Or

If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that :

$$x^2y_2 + xy_1 + y = 0$$

26. Integrate the rational fraction :

$$\frac{2x-3}{(x^2+1)(2x+3)}$$

(/)

Or

Using the properties of definite integrals evaluate :

$$\int_{-5}^{5} |x+2|\,dx$$

27. Find the general solution of the differential equation :

$$x\frac{dy}{dx}+2y=x^2\log x$$

Or

Show that the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is homogeneous and solve it.

28. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2, 2, 1)$.

Or

Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

29. Find the probability distribution of number of doublets in three throws of a pair of dice.

Or

Two balls are drawn at random with replacement from a box containing 10 black balls and 8 red balls. Find the probability that :

(a) Both balls are red

(b) First ball is black and second is red

(c) One of them is black and other is red

A–6–B

Roll No. .....

Total No. of Questions : 29] [Total No. of Printed Pages : 8

12thSZARJD22

6006–B

MATHEMATICS

Time : 2.30 Hours] **[Maximum Marks : 100**

Section–A

(Multiple Choice Questions) 1 eac

1. The relation R in the set {1, 2, 3} given by R = {(x, y) | x > y x, y ∈ A} is :

(A) Reflexive (B) Symmetric

(C) Transitive (D) None of these

2. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is :

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{4}$

Turn O

If A, B are symmetric matrices of same order, then AB – BA is a :

(A) Symmetric matrix

(B) Skew symmetric matrix

(C) Zero matrix

(D) Identity matrix

. The direction cosines of a unit vector along z-axis are :

(A) (1, 0, 0)

(B) (0, 1, 0)

(C) (0, 0, 1)

(D) (1, 1, 1)

Section–B

(Very Short Answer Type Questions) 2 each

5. Find x and y if :

$$2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$$

6. Evaluate :

$$\int(4e^{3x}+1)dx$$

7. Verify that the function $y = x^2 + 2x + C$ is the solution of the differential equation $\frac{dy}{dx}-2x-2=0$.

8. Find the rate of change of the area of a circle with respect to its radius r when $r = 4$ cm.

9. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

10. Solve the following L.P.P. graphically :

Maximise :

$$Z = 3x + 4y$$

Subject to the constraints :

$$x + y \leq 4,$$

$$x \geq 0,\ y \geq 0$$

11. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find $P\left(\frac{A}{B}\right)$.

12. Given two independent events A and B such that $P(A) = \cdot 3$, $P(B) = \cdot 6$. Find P(A or B).

Section–C

(Short Answer Type Questions) 4 each

13. Given $f : R \to R$ given by $f(x) = 4x + 3$. Show that f is invertible.

    Find the inverse of f.

14. Write the following function in the sumplest form :

$$\tan^{-1}\left[\frac{\cos x - \sin x}{\cos x + \sin x}\right] \quad 0 < x < \pi$$

15. Express the matrix $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of symmetric and a skew-symmetric matrix.

16. Find the relationship between a and b so that the function f defined by :

$$f(x) = \begin{cases} ax + 1 & \text{if} \quad x \leq 3 \\ bx + 3 & \text{if} \quad x > 3 \end{cases}$$

    is continuous at $x = 3$.

17. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is :

    (a) Strictly increasing

    (b) Strictly decreasing

18. Find the equations of the tangent and normal to the given curve at the indicated point $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5).

19. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

20. Find the area of the region bounded by the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

21. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

22. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$. then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

23. Solve the following problem graphically :

Minimise and maximise : $Z = 3x + 9y$

Subject to the linear constraints :

$$x + 3y \leq 60,\ x \leq y,$$

and $$x + y \geq 10,\ x \geq 0,\ y \geq 0$$

Section–D

(Long Answer Type Questions)

24. By using properties of determinants prove that :

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

Or

If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that $A(\text{adj } A) = |A|I$. Also find A^{-1}

25. Find $\frac{dy}{dx}$ of the function $x^y + y^x = 1$.

Or

If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that :

$$x^2y_2 + xy_1 + y = 0$$

26. Integrate the rational fraction :

$$\frac{2x-3}{(x^2-1)(2x+3)}$$

Or

Using the properties of definite integrals evaluate :

$$\int_{-5}^{5} |x+2|\,dx$$

27. Find the general solution of the differential equation :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Or

Show that the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is homogeneous and solve it.

28. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2, 2, 1)$.

Or

Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

). Find the probability distribution of number of doublets in three throws of a pair of dice

Or

Two balls are drawn at random with replacement from a box containing 10 black balls and 8 red balls. Find the probability that

a) Both balls are red

b) First ball is black and second is red

:) One of them is black and other is red

Roll No. .....

A–6–C

Total No. of Questions : 29] [Total No. of Printed Pages : 8

12thSZARJD22

6006–C

MATHEMATICS

Time : 2.30 Hours] [Maximum Marks : 100

Section–A

(Multiple Choice Questions)

1 each

1. Let R be the relation on the set N given by $R = \{(a, b) \mid a = b - 2, b > 6\}$. Choose the correct answer.

(A) $(2, 4) \in R$

(B) $(3, 8) \in R$

(C) $(6, 8) \in R$

(D) $(8, 7) \in R$

2. $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$ is equal to :

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

3. If the matrix A is both symmetric and skew-symmetric, then :

(A) A is a diagonal matrix (B) A is a null matrix

(C) A is a square matrix (D) None of these

4. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ is a non-zero scalar, then $\lambda\vec{a}$ is a unit vector if :

(A) $\lambda = 1$ (B) $\lambda = -1$

(C) $a = |\lambda|$ (D) $a = \frac{1}{|\lambda|}$

Section–B

(Very Short Answer Type Questions) 2 each

5. Find the value of x for which :

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

6. Evaluate :

$$\int (1-x)\sqrt{x}\, dx$$

7. Find the rate of change of area of a circle with respect to its radius r when $r = 6$ cm.

8. Find the general solution of the differential equation :

$$\frac{dy}{dx} = e^{x+y}$$

9. Find angle θ between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

10. Solve the following L.P.P. graphically :

Maximise :

$$Z = 3x + 4y$$

Subject to the constraints :

$$x + y \leq 4,$$

$$x \geq 0,\ y \geq 0$$

11. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P(not A and not B).

12. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, evaluate $P\left(\frac{A}{B}\right)$.

Section–C

(Short Answer Type Questions) 4 each

13. If

$$f(x)=\frac{4x+3}{6x-4} \quad x \neq \frac{2}{3},$$

show that $f \circ f = x$ for all $x \neq \frac{2}{3}$. What is the inverse of f ?

14. Prove that :

$$\tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2} = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

15. If $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that :

$$A^2 - 5A + 7I = 0$$

16. Find the relationship between a and b so that the function f defined by :

$$f(x)=\begin{cases} ax+1 & \text{if} \quad x \leq 3 \\ bx+3 & \text{if} \quad x > 3 \end{cases}$$

is continuous at $x = 3$.

17. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is :

(a) Strictly increasing

(b) Strictly decreasing

18. Find the equations of the tangent and normal to the given curve at the indicated point $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5).

19. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

20. Find the area of the region bounded by the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

21. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

22. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

23. Solve the following problem graphically :

Minimise and maximise : $Z = 3x + 9y$

Subject to the linear constraints :

$$x + 3y \leq 60,\ x \leq y,$$

and $x + y \geq 10,\ x \geq 0,\ y \geq 0$

(Long Answer Type Questions) 6 each

24. By using properties of determinants prove that :

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

Or

If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that A(adj A) = | A |I. Also find A^{-1}.

25. Find $\frac{dy}{dx}$ of the function $x^y + y^x = 1$.

Or

If $y = 3 \cos(\log x) + 4 \sin(\log x)$, show that :

$$x^2y_2 + xy_1 + y = 0$$

26. Integrate the rational fraction :

$$\frac{2x-3}{(x^2-1)(2x+3)}$$

Or

Using the properties of definite integrals evaluate :

$$\int_{-5}^{5} |x+2|\,dx$$

27. Find the general solution of the differential equation :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Or

Show that the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is homogeneous and solve it.

28. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2, 2, 1)$.

Or

Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

29. Find the probability distribution of number of doublets in three throws of a pair of dice.

Or

Two balls are drawn at random with replacement from a box containing 10 black balls and 8 red balls. Find the probability that :

(a) Both balls are red

(b) First ball is black and second is red

(c) One of them is black and other is red

B–5–C

Roll No.....

Total No. of Questions : **29**] [Total No. of Printed Pages : **8**

XIIKDAR21

5005–C

MATHEMATICS

Time : 3 Hours] **[Maximum Marks : 100**

Section–A

(Multiple Choice Questions) 1 each

1. Let R be the relation in the set N given by R = $\{(a, b) \mid a = b - 2, (b > 6)\}$:

   (A) $(2, 4) \in R$ (B) $(3, 8) \in R$

   (C) $(6, 8) \in R$ (D) $(8, 7) \in R$

   (Choose the correct answer)

2. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to :

   (A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$

   (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$ (Choose the correct answer)

3. A and B are symmetric matrices of same order, then AB – BA is a :

(A) Skew symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix (Choose the correct answer)

4. The value of :

$$\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{k}\times\hat{i})+\hat{k}\cdot(\hat{j}\times\hat{i})$$

is :

(A) 0 (B) –1

(C) 1 (D) 3 (Choose the correct answer)

Section–B

(Very Short Answer Type Questions)

5. Evaluate :

$$\int\frac{\sin(\tan^{-1}x)dx}{1+x^2}$$

6. Evaluate :

$$\int_0^1\frac{dx}{\sqrt{1-x^2}}$$

7. Find the values of x, y and z from the equation :

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

8. Find the order and degree of differential equation :

$$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 4$$

9. If a line has direction ratios 2, –1, 2, determine its direction cosines.

10. Define objective function and optimal solution of L.P.P.

11. $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$, $P(A \cup B) = \frac{7}{11}$, find $P(A \cap B)$.

12. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $(A \cap B)$ if A and B are independent events.

Section–C

(Short Answer Type Questions) 4 each

13. If :

$$f(x) = \frac{4x+3}{6x-4} \quad x \neq \frac{2}{3}.$$

Show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. What is the inverse of f ?

14. Write in the simplest form :

$$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$$

15. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

16. Find the equation of tangent and normal to the curve at the indicated point $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0, 5)$.

17. Use differentials, find the approximate value up to 3 places of decimal $(25)^{1/3}$.

18. Integrate the function $x \tan^{-1} x$.

19. Find the relationship between a and b such that the function f defined by :

$$f(x) = \begin{cases} ax + 1 & x \leq 3 \\ bx + 3 & x > 3 \end{cases}$$

is continuous at $x = 3$.

20. If $y = \sin^{-1} x$, show that :

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

21. Find the unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

22. Find the angle between two planes :

$$2x + y - 2z = 5$$

and

$$3x - 6y - 2z = 7$$

using vector method.

23. Solve graphically (L.P.P.) :

Maximize :

$$Z = 4x + y$$

Subject to the constraints :

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0,\ y \geq 0$$

Section–D

(Long Answer Type Questions)

6 each

24. Using the properties of determinants. Prove that :

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

Or

Solve the system of equations by matrix method :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

25. If $y = x^{\sin x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$.

Or

Show that of all the rectangles inscribed in a given fixed circle the square has the maximum area.

26. Find :

$$\int \frac{(3\sin x - 2)\cos x \, dx}{5 - \cos^2 x - 4\sin x}$$

Or

Find the area of the region bounded by the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

27. Find the general solution of the differential equation given by :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Or

Find the general solution of the differential equation :

$$\frac{dy}{dx} = \frac{x+1}{2-y}, \quad y \neq 2$$

28. Find the equation of the plane passing through three points (1, 1, 0) (1, 2, 1), (–2, 2, –1).

Or

Find the shortest distance between the lines whose vector equations are :

$$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$$

and

$$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

29. Find the probability distribution of the number of doublets in three throws of a pair of dice.

Or

A die is thrown 6 times if "getting an odd number" is a success. What is the probability of :

(i) 5 success

(ii) at least 5 success

(iii) at most 5 success

B–5–A

Roll No.....

Total No. of Questions : 26] [Total No. of Printed Pages : 8

XIISZRJDF20 1105–A MATHEMATICS

Time : 3 Hours] [Maximum Marks : 100

(Long Answer Type Questions) 6 each

1. Using properties of determinants, show that :

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x + y + z)^3$$

Or

Solve the following system of equations by matrix method :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

2. Differentiate the function

$$x^{\sin x} + (\sin x)^{\cos x}$$

Or

If $y = 3 \cos (\log x) + 4 \sin (\log x)$, show that :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

3. Evaluate :

$$\int \frac{\cos x \, dx}{(1 - \sin x)(2 - \sin x)}$$

Or

Find $\int_0^2 (x^2 + 1)\, dx$ as the limit of a sum.

4. Show that the differential equation $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$ is homogeneous and solve it.

Or

Find the general solution of the differential equation :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

5. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

and $$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Or

Find the angle between the planes whose vector equations are :

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$$

and $$\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$$

(Short Answer Type Questions) 4 each

6. Find the value of K so that the function "f" defined by :

$$f(x) = \begin{cases} Kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$.

7. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $f \circ f(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of 'f' ?

8. If $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$, then find the value of x.

9. Using the properties of determinants and without expanding, prove that :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

10. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

11. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by :

$$1 - P(A')\ P(B')$$

Prove it.

12. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

13. Find $\left|\vec{a}\times\vec{b}\right|$, if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$.

14. Evaluate :

$$\int_0^1 \frac{x\,dx}{x^2+1}$$

15. Solve the following linear programming problems graphically :

Maximise : $z = 5x + 3y$

Subject to :

$$3x + 5y \leq 15$$

$$5x + 2y \leq 10$$

$$x \geq 0,\ y \geq 0$$

(Very Short Answer Type Questions) 2 each

16. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2}<x<\frac{3\pi}{2}$ in the simplest form.

17. Find x and y, if :

$$2\begin{pmatrix}1 & 3\\ 0 & x\end{pmatrix}+\begin{pmatrix}y & 0\\ 1 & 2\end{pmatrix}=\begin{pmatrix}5 & 6\\ 1 & 8\end{pmatrix}$$

18. Find $\frac{dy}{dx}$, if $2x + 3y = \sin x$.

19. Find $\frac{dy}{dx}$, if $x = 2at^2$, $y = at^4$.

20. The volume of a cube is increasing at the rate of 8 cm^3/s. How fast is the surface area increasing when the length of an edge is 12 cm ?

21. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

22. Shade the feasible region of L.P.P. $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

23. Compute P(A/B), if P(B) = 0.5 and P(A $\cap$ B) = 0.32.

24. Evaluate :

$$\int x \sin x \, dx$$

25. Evaluate :

$$\int_0^1 \frac{dx}{1+x^2}$$

(Objective Type Questions) 1 each

26. (i) Define Identity Matrix.

(ii) Area of the region bounded by the curve $y^2 = 4x$, y–axis and the line $y = 3$ is $\frac{9}{4}$. (True/False)

(iii) The degree of the differential equation $\frac{dy}{dx} - \cos x = 0$ is 0.

(True/False)

(iv) $\frac{d}{dx}(\sin^2 x + \cos^2$ = ... (Fill in the blank)

(v) $\int \sqrt{a^2 + x^2}\, dx$ = ... (Fill in the blank)

(vi) The direction cosines of x-axis are ..... .

(Fill in the blank)

(vii) The rate of change of the area of a circle with respect to its radius 'r', when $r = 5$ cm is :

(A) 8π cm^2/cm (B) 10π cm^2/cm

(C) 12π cm^2/cm (D) None of these

(viii) The slope of tangent to the curve $y = x^3 - x$ at $x = 2$ is :

(A) 11 (B) 12

(C) 13 (D) None of these

(ix) $\int x^2 e^{x^3} dx$ is equal to :

(A) $\frac{1}{3}e^{x^3}+C$ (B) $\frac{1}{3}e^{x^2}+C$

(C) $\frac{1}{2}e^{x^3}+C$ (D) $\frac{1}{2}e^{x^2}+C$

(x) Integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is

(A) x (B) x^2

(C) x^3 (D) None of these

B–5–B

Roll No.....

Total No. of Questions : 26] [Total No. of Printed Pages : 8

XIISZRJDF20

1105–B

MATHEMATICS

Time : 3 Hours] [Maximum Marks : 100

(Long Answer Type Questions) 6 each

1. Using properties of determinants, show that :

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x + y + z)^3$$

Or

Solve the following system of equations by matrix method :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

2. Differentiate the function :

$$x^{\sin x} + (\sin x)^{\cos x}$$

Or

If $y = 3 \cos (\log x) + 4 \sin (\log x)$, show that :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

3. Evaluate :

$$\int \frac{\cos x \, dx}{(1 - \sin x)(2 - \sin x)}$$

Or

Find $\int_0^2 (x^2 + 1)\, dx$ as the limit of a sum.

4. Show that the differential equation $(x^2 + xy)\, dy = (x^2 + y^2)\, dx$ is homogeneous and solve it.

Or

Find the general solution of the differential equation :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

5. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

and $$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Or

Find the angle between the planes whose vector equations are :

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$$

and $$\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$$

(Short Answer Type Questions) 4 each

6. Find the value of K so that the function "f" defined by :

$$f(x) = \begin{cases} Kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$.

7. If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, show that $f \circ f(x) = x$, for all $x \neq \dfrac{2}{3}$. What is the inverse of 'f' ?

8. If $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$, then find the value of x.

9. Using the properties of determinants and without expanding, prove that :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

10. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

11. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by :

$$1 - P(A')\ P(B')$$

Prove it.

12. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

13. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

14. Evaluate :

$$\int_0^1 \frac{x\,dx}{x^2+1}$$

15. Solve the following linear programming problems graphically :

Maximise : $Z = 5x + 3y$

Subject to

$$3x + 5y \leq 15$$

$$5x + 2y \leq 10$$

$$x \geq 0,\ y \geq 0$$

(Very Short Answer Type Questions) 2 each

16. Express $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), x \neq 0$ in the simplest form.

17. Find the value of x and y, if :

$$x\begin{pmatrix}2\\3\end{pmatrix} + y\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}10\\5\end{pmatrix}$$

18. Find $\frac{dy}{dx}$, if $2x + 3y = \sin y$.

19. Find $\frac{dy}{dx}$, if $x = a \sec \theta$, $y = b \tan \theta$.

20. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

21. Find the angle between the vectors $\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$.

22. Shade the feasible region of L.P.P. :

$x + 2y \leq 10$, $3x + y \leq 15$, $x, y \geq 0$.

23. If P(A) = 0.8, P(B) = 0.5 and P(B/A) = 0.4 find P(A/B).

24. Evaluate :

$$\int x \sin 3x \, dx$$

25. Evaluate :

$$\int_2^3 \frac{dx}{x^2 - 1}$$

(Objective Type Questions) 1 each

26. (i) Define skew-symmetric matrix.

(ii) Area lying between the curves $y^2 = 4x$ and $y = 2x$ is $\frac{1}{3}$.

(True/False)

(iii) $\int(\text{cosec}^2 x - \cot^2 x) =$ ..... (Fill in the blank)

(iv) The degree of the differential equation

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 0$$ is 2. (True/False)

(v) $\int\sqrt{a^2 - x^2}$ = ... (Fill in the blank)

(vi) Direction cosines of y-axis are ..... . (Fill in the blank)

(vii) The rate of change of the area of a circle with respect to its radius 'r', when $r = 3$ cm is :

(A) 4 cm^2/cm (B) 6 cm^2/cm

(C) 8 cm^2/cm (D) None of these

(viii) The slope of tangent to the curve $y = 3x^4 - 4x$ at $x = 4$ is :

(A) 760 (B) 762

(C) 764 (D) None of these

Turn Over

(ix) $\int e^x \sec x (1+\tan x)\, dx$ is equal to :

(A) $e^x \cos x + C$ (B) $e^x \sec x + C$

(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

(x) The integrating factor of the differential equation $y\, dy - (x + 2y^2)\, dx = 0$ is :

(A) $\frac{1}{y}$ (B) $\frac{1}{y^2}$

(C) $\frac{1}{x}$ (D) None of these

B–5–C

Roll No.....

Total No. of Questions : 26] [Total No. of Printed Pages : 8

XIISZRJDF20

1105–C

MATHEMATICS

Time : 3 Hours] [Maximum Marks : 100

Long Answer Type Questions) 6 each

1. Using properties of determinants, show that :

$$\begin{vmatrix} x+y & z & x \\ z & y+z+x & y \\ z & x & z+x+y \end{vmatrix} = 2(x+y+z)^3$$

Or

Solve the following system of equations by matrix method :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

2. Differentiate the function :

$$x^{\sin x} + (\sin x)^{\cos x}$$

Or

If $y = 3 \cos (\log x) + 4 \sin (\log x)$, show that :

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

3. Evaluate :

$$\int \frac{\cos x \, dx}{(1 - \sin x)(2 - \sin x)}$$

Or

Find $\int_0^2 (x^2 + 1) \, dx$ as the limit of a sum.

4. Show that the differential equation $(x^2 + xy) \, dy = (x^2 + y^2) \, dx$ is homogeneous and solve it.

Or

Find the general solution of the differential equation :

$$x \frac{dy}{dx} + 2y = x^2 \log x$$

5. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Or

Find the angle between the planes whose vector equations are :

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$$

$$\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$$

(Short Answer Type Questions) 4 each

6. Find the value of K so that the function "f" defined by :

$$f(x) = \begin{cases} Kx + 1, & \text{if } x \leq 5 \\ 3x - 5, & \text{if } x > 5 \end{cases}$$

is continuous at $x = 5$.

7. If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, show that $f \circ f(x) = x$, for all $x \neq \dfrac{2}{3}$ What is the inverse of 'f' ?

8. If $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$, then find the value of x.

9. Using the properties of determinants and without expanding, prove that :

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

10. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

11. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by :

$$1 - P(A')\ P(B')$$

Prove it.

12. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

13. Find $\left|\vec{a}\times\vec{b}\right|$, if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$.

14. Evaluate :

$$\int_0^1 \frac{x\,dx}{x^2+1}$$

15. Solve the following linear programming problems graphically :

Maximise : $Z = 5x + 3y$

Subject to :

$$3x + 5y \leq 15$$

$$5x + 2y \leq 10$$

$$x \geq 0,\ y \geq 0$$

(Very Short Answer Type Questions) 2 each

16. Express $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$, $0 < x < \pi$, in the simplest form.

17. Find the value of x and y, if :

$$2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix}+\begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix}=\begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$$

18. Find $\frac{dy}{dx}$, if $ax + by^2 = \cos y$.

19. Find $\frac{dy}{dx}$, if $x = \sin t$, $y = \cos 2t$.

20. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long ?

21. Evaluate the product :

$$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$$

22. Shape the feasible region of L.P.P. $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$.

23. Evaluate $P(A \cup B)$, if $2\,P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

24. Evaluate :

$$\int x^2 e^x dx$$

25. Evaluate :

$$\int_2^3 \frac{x\,dx}{x^2+1}$$

(Objective Type Questions) 1 each

26. (i) Define symmetric matrix.

(ii) Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the line $y = 3$ is π. (True/False)

(iii) The degree of the differential equation $\frac{dy}{dx} + 5y = 0$ is 2. (True/False)

(iv) $\frac{d}{dx}(\sec^2 x - \tan^2 x)$ = ... (Fill in the blank)

(v) $\int \sqrt{x^2 - a^2}\, dx$ = ... (Fill in the blank)

(vi) Direction cosines of z-axis are ..... . (Fill in the blank)

(vii) The rate of change of the area of a circle with respect to its radius r, when $r = 2$ cm is :

(A) 8π cm^2/cm (B) 4π cm^2/cm

(C) 2π cm^2/cm (D) None of these

(viii) The slope of tangent to the curve $y = 5x^2 - 6x$ at $x = 2$ is :

(A) 12 (B) 14

(C) 16 (D) None of these

(ix) $\int \tan x \, dx$ is equal to :

(A) log (cos x) + C (B) log (sec x) + C

(C) log (sin x) + C (D) log (cot x) + C

(x) Integrating factor of the differential equation $\frac{dy}{dx} - y = \cos x$ is :

(A) – sin x (B) e^{-x}

(C) e^x (D) None of these

E–5–X

Roll No.....

Total No. of Questions : 26]

[Total No. of Printed Pages : 7

XIIWZJDRO/N19

24905–X

MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 100

(Long Answer Type Questions) 6 each

1. Using properties of determinants prove that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Or

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations :

$$2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3$$

2. Find $\frac{dy}{dx}$ if $y=(\log x)^x+x^{\log x}$.

Or

If $y = (\tan^{-1} x)^2$ show that :

$$(x^2 + 1)^2 y_2 + 2x (x^2 + 1) y_1 = 2$$

3. Evaluate $\int_0^2 (x^2+1)\,dx$ as the limit of a sum.

Or

Evaluate :

$$\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$$

4. Find the equation of the plane passing through three points (1, 1, 0), (1, 2, 1) and (–2, 2, –1).

Or

Find the shortest distance between the lines whose vector equations are :

$$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$$

and $$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}.$$

5. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Or

Find the variance and S.D. of a number obtained on throwing an unbiased die.

(Short Answer Type Questions) 4 each

6. If $f(x)=\frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $(f \circ f) = x$ for all $x \in \neq \frac{2}{3}$. What is f^{-1} ?

7. Prove that :

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$$

8. If $A=\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, verify that A. adj A = $|A| \cdot I$

9. Find $\frac{dy}{dx}$ when :

$$x = a(\cos \theta + \theta \sin \theta)$$

$$y = a(\sin \theta - \theta \cos \theta)$$

10. Find the equation of a tangent to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$.

11. Find the value of 'k' so that the function $f(x)$ defined by :

$$f(x)=\begin{cases} kx+1, & \text{if} \quad x \leq 5 \\ 3x-5, & \text{if} \quad x>5 \end{cases}$$

is continuous at $x = 5$.

12. Find a vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

13. Evaluate :

$$\int_0^{\pi/2} \sin^3 x \, dx$$

14. Solve the homogeneous differential equation :

$$(x^2 - y^2)\, dx + 2xy\, dy = 0$$

15. Solve graphically to :

Maximize :

$$Z = 3x + 2y$$

Subject to constraints :

$$x + 2y \leq 10$$

$$3x + y \leq 15$$

$$x, y \geq 0$$

(Very Short Answer Type Questions) 2 each

16. For the matrix $A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ verify $A'A = I$.

17. Find the rate of change of the area of a circle with respect to its radius r when $r = 3$ cm.

18. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

19. Find $\frac{dy}{dx}$ if $xy + y^2 = \tan x + y$.

20. Evaluate :

$$\int \frac{1-\cos x}{1+\cos x}\,dx$$

21. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \sqrt{4-y^2},\ |y|<2$$

22. Find the unit vector in the direction of PQ, where P and Q are points (1, 2, 3) and (4, 5, 6) respectively.

23. Find the angle between the vectors $\vec{a}$ and $\vec{b}$ with magnitude 1 and 2 respectively and $\vec{a} \cdot \vec{b} = 1$.

24. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 3\hat{k}$.

25. Compute P(A/B) if P(B) = 0.5 and P(A ∩ B) = 0.32

(Objective Type Questions) 1 each

26. (i) Define a symmetric matrix.

(ii) What is linear programme problem ?

(iii) Define Feasible region of LPP.

(iv) Define a symmetric relation.

(v) A die is rolled once. The probability of obtaining even prime number is ..... . (Fill in the blank)

(vi) The direction cosines of x-axis are ..... .

(Fill in the blank)

(vii) A coin is tossed thrice, then probability of exactly 2 heads is ..... .

(Fill in the blank)

(viii) $\frac{d}{dx}(\log \sec x) = \sec x$.

(True/False)

(ix) $\cos\left(\frac{\pi}{6} - \sin^{-1}\frac{1}{2}\right)$ is :

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$ (D) 1

(x) $\int \tan x \, dx =$ ..... .

(Fill in the blank)

E–5–Z

Roll No.....

Total No. of Questions : 26] [Total No. of Printed Pages : 7

XIIWZJDRO/N19

24905–Z

MATHEMATICS

Time : 3 Hours] [Maximum Marks : 100

(Long Answer Type Questions) 6 each

1. Using properties of determinants prove that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Or

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations :

$$2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3$$

2. Find $\frac{dy}{dx}$ if $y=(\log x)^x+x^{\log x}$.

Or

If $y = (\tan^{-1} x)^2$ show that :

$$(x^2 + 1)^2 y_2 + 2x (x^2 + 1) y_1 = 2$$

3. Evaluate $\int_0^2 (x^2+1)\,dx$ as the limit of a sum.

Or

Evaluate :

$$\int \frac{x \cos^{-1} x}{\sqrt{1-x^2}}\,dx$$

4. Find the equation of the plane passing through three points (1, 1, 0), (1, 2, 1) and (–2, 2, –1).

Or

Find the shortest distance between the lines whose vector equations are :

$$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$$

and

$$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}.$$

5. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Or

Find the variance and S.D. of a number obtained on throwing an unbiased die.

(Short Answer Type Questions) 4 each

6. If $f(x)=\frac{4x+3}{6x-4}, x\neq\frac{2}{3}$, show that $(f\circ f)_x = x$ for all $x\neq\frac{2}{3}$. What is inverse of f ?

7. Prove that :

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\ \frac{\pi}{4}\right)$$

8. If $A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$, verify that A. adj A = $|A|\cdot I$

9. Find $\frac{dy}{dx}$ when :

$$x = a(\cos\theta + \theta\sin\theta)$$

$$y = a(\sin\theta - \theta\cos\theta)$$

10. Find the equation of a tangent to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$.

11. Find the value of 'k' so that the function $f(x)$ defined by :

$$f(x)=\begin{cases} kx+1, & \text{if } x\leq 5 \\ 3x-5, & \text{if } x>5 \end{cases}$$

is continuous at $x = 5$.

12. Find a vector of magnitude 5 units and parallel to the resultant of vectors $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

13. Evaluate :

$$\int_0^{\pi/2} \sin^3 x \, dx$$

14. Solve the homogeneous differential equation :

$$(x^2 - y^2)\, dx + 2xy\, dy = 0$$

15. Solve graphically to :

Maximize :

$$Z = 3x + 2y$$

Subject to constraints :

$$x + 2y \leq 10$$

$$3x + y \leq 15$$

$$x, y \geq 0$$

(Very Short Answer Type Questions) 2 each

16. For the matrix $A=\begin{bmatrix}1 & 5\\ 6 & 7\end{bmatrix}$, verify that $(A - A')$ is skew symmetric.

17. Find the rate of change of the area of a circle with respect to its radius r when $r = 5$ cm.

18. Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta=\frac{\pi}{4}$.

19. Find $\frac{dy}{dx}$ if $\sin^2 x + \cos^2 y = 1$.

20. Evaluate :

$$\int\frac{\sin^2 x}{1+\cos x}dx$$

21. Find the general solution of the differential equation :

$$\frac{dy}{dx}+y=1,\quad y\neq 1$$

22. Find a vector in the direction of vector $5\hat{i}-\hat{j}+2\hat{k}$ which has magnitude 8 units.

23. Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$.

24. Find $|\hat{a}\times\hat{b}|$, where $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$.

25. Given P(A) = 0·8, P(B) = 0·5 and P(B/A) = 0·4. Compute P(A ∩ B).

(Objective Type Questions) 1 each

26. (i) Define a scalar matrix.

(ii) Define feasible region of linear inequations.

(iii) Where do the optimum value points exist ?

(iv) Define an equivalence relation.

(v) The probability of obtaining doublets when a pair of dice is rolled is ..... . (Fill in the blank)

(vi) The direction cosines of z-axis are ..... .

(Fill in the blank)

(vii) A coin is tossed thrice, then probability of getting exactly one head is ..... . (Fill in the blank)

(viii) $\frac{d}{dx}(\log e^x) =$ ..... . (Fill in the blank)

(ix) The value of $\sin\left(\frac{\pi}{3} - \cos^{-1}\frac{1}{2}\right)$ is :

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{2}$

(Choose the correct one)

(x) $\int \sec x\, dx$ is equal to :

(A) $\log|\sec x + \tan x| + C$

(B) $\log|\sec x - \tan x| + C$

(C) $\log|\tan x - \sec x| + C$

(D) None of these (Choose the correct one)

A–3–X

Roll No.

Total No. of Questions : 20] [Total No. of Printed Pages : 7 + Graph

SSERS/ZJM18

17003-X

MATHEMATICS

Time : 3 Hours] **[Maximum Marks : 100**

Note :– Attempt all questions.

1. (i) $\sqrt{121}$ is :

(a) an even number (b) an irrational number

(c) a rational number (d) None of these

(ii) If an event cannot occur, then its probability is :

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) None of these

(iii) The common difference of an A.P :

$a, a + 2d, a + 4d, a + 6d, \ldots\ldots\ldots\ldots$

is :

(a) d (b) $-2d$

(c) $2d$ (d) None of these

(iv) Volume of cone is :

(a) $\frac{2}{3}\pi r^2 h$ (b) $\frac{1}{3}\pi r^2 h$

(c) $\pi r l$ (d) None of these

(v) How many tangents can be constructed from a point inside the circle ?

(a) 0 (b) 2

(c) Infinite (d) None of these

(vi) The zeroes of the polynomial $9x^2 - 5$ are :

(a) $\frac{5}{3}, \frac{-5}{3}$ (b) $\frac{3}{\sqrt{5}}, \frac{-3}{\sqrt{5}}$

(c) $\frac{\sqrt{5}}{3}, \frac{-\sqrt{5}}{3}$ (d) None of these 1×6=6

2. Find the distance between the points (–5, 7) and (–1, 3). 2

3. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°. 2

4. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find its radius. 2

5. Find the sum of the first 15 multiples of 8. 4

6. Check whether 6^n can end with the digit 0 for any natural number n. 4

7. Solve the following pair of equations by elimination method :

$$3x + 4y = 10$$

$$2x - 2y = 2$$ 4

8. Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Find Aftab's present age. 4

9. Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$. 4

10. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting :

(i) a red face card

(ii) the jack of hearts 4

11. Find the roots of $4x^2 + 3x + 5 = 0$ by the method of completing the square.

Or

Find the value of k for which the following quadratic equation has two equal roots

$$kx(x - 2) + 6 = 0$$ 6

12. A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the speed of the train.

Or

Find the roots of the quadratic equation $100x^2 - 20x + 1 = 0$ by factorisation. 6

13 If the areas of two similar triangles are equal, prove that they are congruent.

Or

In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9AD^2 = 7AB^2$. 6

14. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. Prove it.

Or

If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that :

$$\frac{AB}{PQ} = \frac{AD}{PM}$$ 6

15. Find the co-ordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

Or

Find the value of '*k*', for which the points (8, 1), (*k*, –4) and (2, –5) are collinear. 6

16. If $\cot\theta = \frac{7}{8}$, evaluate :

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

Or

Prove that :

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A} = 2 \sec A$$ 6

17. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m Find the height of the tree.

Or

If $\sin (A - B) = \frac{1}{2}$, $\cos (A + B) = \frac{1}{2}$, $0^\circ < A + B \leq 90^\circ$, $A > B$, find A and B. 7

18. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that :

$$\angle PTQ = 2\ \angle OPQ$$

Or

Prove that the tangents drawn at the ends of a diameter of a circle are parallel. 7

19. Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Or

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm. 7

20. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have ? Find the surface area of the solid.

Or

The slant height of a frustum of cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum. 7

F–5–Y

Roll No.....

Total No. of Questions : 26]

[Total No. of Printed Pages : 8

12thRWZJO18

20105–Y

MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 100

(Long Answer Type Questions)

6 each

1. Using properties of determinants prove that :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc.$$

Or

Solve system of linear equations, using matrix method :

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

2. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

Or

If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$.

3. Evaluate given definite integral as limit of sum :

$$\int_0^4 (x + e^{2x})\,dx$$

Or

Integrate the rational function :

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

4. Find the shortest distance between the lines :

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Or

Find the distance of a point (2, 5, –3) from the plane :

$$\vec{r}.\left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$$

5. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X ? Is X a random variable ?

Or

Bag I contains 3 red and 4 black balls and Bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bag and is found to be red. Find the probability that it was drawn from Bag II.

(Short Answer Type Questions) 4 each

6. If :

$$f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3},$$

show that f o $f(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f ?

7. Write the function in the simplest form :

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0; \frac{-a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$$

8. Prove that any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrix.

9. Find $\frac{dy}{dx}$ in $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$.

10. Find the equation of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}$, $x \neq 3$.

11. Find both the maximum value and minimum value of $3x^4 - 8x^3 + 12x^2 - 48x - 25$ on the interval $[0, 3]$.

12. Using integration, find the area of the triangular region whose sides have the equations $y = 2x + 1$; $y = 3x + 1$; $x = 4$.

13. Show that the differential equation is homogeneous and solve it :

$$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

14. Find the co-ordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$.

15. Solve the Linear Programming problem graphically :

Minimize $z = 3x + 5y$

such that :

$$x + 3y \geq 3$$

$$x + y \geq 2$$

$$x, y \geq 0$$

(Very Short Answer Type Questions) 2 each

16. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27\ |A|$.

17. Examine the function for continuity $f(x) = x - 5$.

18. Evaluate :

$$\int \frac{dx}{\sqrt{2x - x^2}}$$

19. Evaluate :

$$\int_{-1}^{1} (x+1)\,dx$$

20. Verify that the given function is a solution of the corresponding differential equation :

$$y = x^2 + 2x + c$$

$$y' - 2x - 2 = 0$$

21. Find the the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

22. Write two different vectors having same direction.

23. Find the unit vector in the direction of a vector $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.

24. Minimize $Z = 3x + 2y$ subject to the constraints $x + y \geq 8$, $x, y \geq 0$.

25. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

(Objective Type Questions) 1 each

26. (i) Define transitive relation.

(ii) Find the value of :

$$\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$$

(iii) If A and B are two skew-symmetric matrices of same order, then AB is symmetric matrix of ..... . (Fill in the blank)

(iv) The derivative of sinx w.r.t. cos x is ..... .

(Fill in the blank)

(v) $\int \frac{\sin^6 x}{\cos^8 x} dx = \frac{\tan^7 x}{7} + c$ (True/False)

(vi) $\int_{-a}^{a} f(x)\,dx = 0$, if f is an odd function. (True/False)

(vii) The order of the differential equation of all circles of given radius a is :

(a) 1 (b) 2

(c) 3 (d) 4

(viii) The value of λ for which the two vectors $2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\lambda\hat{j}+\hat{k}$ are perpendicular is :

(a) 2 (b) 4

(c) 6 (d) 8

(ix) If $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{5}$, then $P(B/A) + P(A/B)$ equals :

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{5}{12}$ (d) $\frac{7}{2}$

(x) For the following probability distribution :

X	-4	-3	-2	-1	0
P(X)	0.1	0.2	0.3	0.2	0.2

E(X) is equal to :

(a) 0 (b) -1

(c) -2 (d) -1.8

Roll No.....

G–22–C

Total No. of Questions : **26]** [Total No. of Printed Pages : **8 + Graph**

XIIRJSZM17
12622-C
MATHEMATICS

Time : 3 Hours] **[Maximum Marks : 100**

(Long Answer Type Questions) 6 each

1. Using properties of determinants, show that :

$$\begin{vmatrix} (y+z)^2 & xy & xz \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

Or

Solve the system of the following equations using matrix method :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

2. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$.

Or

If $y = (\tan^{-1} x)^2$ show that :

$$(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$$

3. Evaluate :

$$\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)dx$$

Or

Evaluate :

$$\int \frac{(3\sin\phi - 2)\cos\phi}{5 - \cos^2\phi - 4\sin\phi} d\phi$$

4. Find the equation of the plane that contains the point (1, –1, 2) and is perpendicular to each of the plane $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$.

Or

Lt $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 15$.

5. A die is thrown six times. If getting an odd number is a success. What is the probability of getting :

(i) 5 successes

(ii) at least 5 successes

(iii) at most 5 successes ?

Or

Two cards are drawn successively with replacement from a well suffled pack of 52 cards. Find the probability distribution of the number of aces.

(Short Answer Type Questions) 4 each

6. Consider $f : R \to [4, \infty]$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$ where R is the set of all non-negative real numbers.

7. Prove that :

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right); \quad xy > -1$$

8. Find points of all discontinuity of f where f is defined by

$$f(x) = \begin{cases} x+1 & \text{if} \quad x \geq 1 \\ x^2+1 & \text{if} \quad x < 1 \end{cases}$$

9. Find the equation of the Tangent line to the curve $y = x^2 - 2x + 7$ which is :

   (a) Parallel to the line $2x - y + 9 = 0$

   (b) Perpendicular to the line $5y - 15x = 13$.

10. At what points in the interval $[0, 2\pi]$ does the function $\sin 2x$ attain its maximum value ?

11. Solve the differential equation :

$$\cos^2 x \frac{dy}{dx} + y = \tan x \left(0 \le x \le \frac{\pi}{2}\right)$$

12. Using integration find the area of region bounded by the triangle whose vertices are (1, 0); (2, 2) and (3, 1).

13. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

14. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$; then find the value of $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$.

15. Maximize and Minimize

$$Z = 5x + 10y$$

Subject to :

$$x + 2y \le 120$$

$$x + y \ge 60$$

$$x - 2y \ge 0;, \; x, \; y \ge 0.$$

(Very Short Answer Type Questions) 2 each

16. If :

$$x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$$

find x and y.

17. Evaluate :

$$\begin{vmatrix}102 & 18 & 36\\1 & 3 & 4\\17 & 3 & 6\end{vmatrix}$$

18. Find g o f and f o g if $f : R \to R$ and $g : R \to R$ given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that $g \circ f \neq f \circ g$.

19. Solve the differential equation :

$$(e^x + e^{-x})\frac{dy}{dx} = (e^x - e^{-x})$$

20. If P(A) = 0.5 and P(B) = 0.8 and P(B/A) = 0.4, find :

(i) P(A/B)

(ii) P(A $\cup$ B)

21. Maximize :

$$Z = 3y + 5x$$

subject to the constraints :

$$3x + 5y \leq 15;$$
$$x, y \geq 0$$

22. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ verify that :

(i) (A + A′) is a symmetric matrix

(ii) (A – A′) is a skew symmetric matrix.

23. Find the general solution of :

$$y \log y dx - x dy = 0$$

24. $\int x^2 \cdot \log x \, dx$.

25. Find $\frac{dy}{dx}$ for $x = a(\theta - \sin\theta)$

$$y = a(1 + \cos\theta)$$

(Objective Type Questions) 1 each

26. Choose the correct answer :

(i) If A and B are invertible matrices of the same order, then (AB)′ is equal to :

(a) A′B′ (b) B′A′

(c) –A′B′ (d) –B′A′

G–3–Y

Roll No.....

Total No. of Questions : 20] [Total No. of Printed Pages : 8 + Graph

SSERJWZN16
11303–Y
MATHEMATICS

Time : 3 Hours] [Maximum Marks : 100

1. Choose the correct answer :

(i) Which of the following pattern of number are in A.P. ?

(a) 31, 27, 23, 18 .....

(b) –3, –6, –9, –12 .....

(c) 2, 7, 9, 11, 13

(d) 2, 4, 6, 10, 16, 24 .....

(ii) A linear equation in two variable represent a :

(a) Point

(b) Circle

(c) Line

(d) None of these

Turn Over

(iii) Volume of sphere is :

(a) $\frac{3}{4}\pi r^3$ (b) $\frac{2}{3}\pi r^2$

(c) $\frac{4}{3}\pi r^3$ (d) $\frac{2}{3}\pi r^3$

(iv) A line intersecting a circle in two points is called ..... .

(v) Probability of an impossible event is ..... .

(vi) Zero of the quadratic polynomial $x^2 - 2x - 8$ is :

(a) -2 and 4 (b) 2 and 4

(c) 2 and –4 (d) –2 and –4 1×6=6

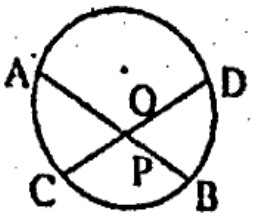
2. Find the distance between the points (0, 0) and (36, 15). 2

3. Evaluate :

$$\frac{\tan 26}{\cot 64^\circ}$$

2

4. In figure given AP = 4 cm, PB = 3 cm and CP 6 cm. Then find PD.



2

5. Find the L.C.M. of 12, 15 and 21 by factorization method. 4

6. The sum of 1st nth term of an A.P. is given by $S_n = 3n^2 - 4n$. Determine 12th term of an A.P. 4

7. Solve the equation by Elimination method :

$$2x - 3y = 4$$

$$x + y = 5$$

4

8. Find the zero of quadratic polynomial and verify relationship between zero and co-efficient of :

$$x^2 - 2x - 8$$

4

Turn Over

9. The sum of the digits of two number is 9. Also nine times this number is twice the number obtained by reversing the order of digits. Find the number. 4

10. One card is drawn from a well suffled deck of 52 cards. Find the probability of getting :

(i) the jack of hearts

(ii) the queen of diamond 4

11. The sum of the reciprocals of Rehman's ages (in years) 3 years ago and 5 years from new is $\frac{1}{3}$. Find the present age.

Or

Find the roots of the quadratic equation by the method of completing the square $2x^2 + x - 4 = 0$. 6

12. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find other two sides.

Or

Find the roots of the following equation :

$$\frac{1}{x} - \frac{1}{x-2} = 3 \quad x \neq 0, 2$$ 6

13. Prove that the ratio of the area of two similar triangle is equal to the square of the ratio of their corresponding medians.

Or

ABCD is a trapezium in which AB || DC and its diagonal intersect each other at the point O. Show that :

$$\frac{AO}{BO} = \frac{CO}{DO}$$ 6

14. D and E are points on the sides CA and CB respectively of a triangle ABC right angles at C. Prove that :

$$AE^2 + BD^2 = AB^2 + DE^2$$

Or

ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel of AB. Show that :

$$\frac{AE}{ED} = \frac{BF}{FC}$$ 6

15. If A and B are (–2, –2) and (2, –4) respectively, find the co-ordinates of P such that AP = $\frac{3}{7}$AB, and P lies on the line segment AB.

Or

Find the area of quadrilateral whose vertices taken in order are (–4, –2), (–3, –5) (3, –2) and (2, 3). 6

16. Evaluate :

$$\frac{5\cos^2 60 + 4\sec^2 30 - \tan^2 45}{\sin^2 30 + \cos^2 30}$$

Or

Prove :

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$ 6

17. The length of a tangent from a point A at distance 5 cm from the centre of a circle is 4 cm. Find the radius of circle.

Or

Prove that the tangents drawn at the ends of a diameter of a circle are parallel.. 7

18. Draw a pair of tangent to a circle of radius 5 cm which are inclined to each other at an angle of 60°. Write steps.

Or

Draw a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC. 7

19. The angle of depression of the top and the bottom of an 8 m tall building from the top of a multistoreyed building are 30° and 45° respectively. Find the height of the multistoreyed building and the distance between the two buildings.

Or

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Find the height of tower. 7

20. How many silver coins, 1·75 cm in diameter and of thickness 2 mm must be melted to form a cuboid of dimensions 5·5 cm × 10 cm × 3·5 cm ?

Or

A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of platform. 7

C–3–P

Roll No.....

Total No. of Questions : 20] [Total No. of Printed Pages : 7

SSEWZJN15

6603–P

MATHEMATICS

Time : 3 Hours] [Maximum Marks : 100

1. Choose the correct answer from the four given answers and write on your answer-book provided to you :

(i) LCM of 135 and 225 is :

(a) 45

(b) 675

(c) 255

(d) None of the above 1

(ii) The common difference of the A.P.

0, –4, –8, –12, ... is :

(a) 2

(b) 0

(c) −4

(d) 4

(iii) The co-efficient of x^2 in the polynomial $x^2 - 2x - 8$ is :

(a) −2

(b) −8

(c) 2

(d) None of the above

(iv) The circle which touches all the sides of a triangle is called

(a) Excircle

(b) Circumcircle

(c) Incircle

(d) None of the above

(v) The circumference of a circle with diameter 21 cm is :

(a) 132 cm

(b) 66 cm

(c) 132 sq cm

(d) None of the above

(vi) A die is rolled once. The probability of getting a prime number is :

(a) 1

(b) 2

(c) 3

(d) None of the above 1

2. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle. 2

3. Find the distance between the pair of points (–5, 7) and (–1, 3). 2

4. Evaluate :

$$\frac{\tan 65^\circ}{\cot 25^\circ}$$ 2

5. Use Euclid's division algorithm to find the HCF of 455 and 42. 4

6. Which term of the A.P. ?

7, 13, 19, ... is 205 4

7. Solve the following pair of equations by cross multiplication method:

$$x - 3y = 7$$

$$3x - 3y = 15$$

8. The difference between two numbers is 26 and one number is three times the other. Find them? ~~1/2 if we only add 1 to the denominator. What is the fraction ?~~ 4

9. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and $\sqrt{5}$ respectively. 4

10. The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.

    (i) What is the probability that on a given day it was correct ?

    (ii) What is the probability that it was not correct on a given day ?

11. Solve the general quadratic equation

$$ax^2 + bx + c = 0$$

Or

The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 30 m more than the shorter side, find the sides of the field.

12. Find the value of 'k' for which the following quadratic equation has two equal roots.

$$kx(x - 2) + 6 = 0$$

Or

The product of two consecutive positive integers is 306. Find the integers. 6

13. Prove that in a right triangle, the square of the hypotenues is equal to the sum of the squares of the other two sides.

Or

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians. 6

14. In a triangle ABC, altitudes AD and CE intersect each other at the point 'P'. Show that :

$$\Delta AEP \sim \Delta ADB$$

$$\Delta PDC \sim \Delta BEC$$

Or

E and F are points on the sides PQ and PR respectively of a ΔPQR. For the following case state whether EF || QR.

PE = 3·9 cm, EQ = 3 cm, PF = 3·6 cm and FR = 2·4 cm. 6

Turn Over

15. Find the value of 'k' for which the points (8, 1), (k, –4) and (2, –5) are collinear.

Or

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y. 6

16. If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Or

If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = \frac{1}{\sqrt{3}}$; $0° < (A + B) \leq 90°$; $A > B$, find A and B. 6

17. Express the trigonometric ratio of sin A, sec A and tan A interms of cot A.

Or

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower. 7

18. Prove that the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Or

Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact. 7

19. Draw a triangle ABC with sides BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$, then construct a triangle whose sides are 3/4 of the corresponding sides of the triangle ABC. (Steps of construction is not required)

Or

Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm. (Steps of construction is not required) 7

20. The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Or

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap. 7

C–23

Roll No .....

Total No of Questions 14

Total No of Printed Pages 4 + Graph

HSIJDT1DXII

56022

MATHEMATICS

(Term-1st)

Time : 2½ Hours] **[Maximum Marks : 35**

Note : In case of failure re-appear and fresh private cases: i.e. candidates appearing for the first time after having passed the Secondary School Examination marks secured out of 35 shall be raised proportionately as if obtained out of 50.

1. Let A = {1, 2, 3 ..... 14} Define a relation R from A to A by R = {(x, y) : $3x - y = 0$ where $x, y \in A$}. Write down its domain, codomain and range

Or

Define Cartesian Product of two sets Let A = {1, 2} and B = {3, 4}. Write A × B. How many subsets will A × B have ? List them. 5

2. Prove that :

$$\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1.$$

Or

Find the general solution of the equation :

$$\sin 2\theta + \cos \theta = 0$$ 5

3. If $Z_1 = 2 - i$, $Z_2 = 1 + i$, find $\left|\frac{Z_1 + Z_2 + 1}{Z_1 - Z_2 + 1}\right|$.

Or

Prove that $\left(\frac{\bar{Z}_1}{Z_2}\right) = \frac{\bar{Z}_1}{\bar{Z}_2}$, where Z_1 and Z_2 are 2 complex numbers. 5

4. Prove that :

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x.$$ 3

5. Define : 3

(i) Identity function.

(ii) Absolute value function.

(iii) Constant function.

6. Calculate the mean deviation from the mean for the following data : 3

x_i	5	10	15	20	25
f_i	7	4	6	3	5

7. Solve Graphically : 2

$$3x + 4y \leq 12.$$

8. Which term of the sequence $\sqrt{3}, 3, 3\sqrt{3}$..... is 729. 2

9. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find P (A or B), if A and B are mutually exclusive events. 2

Choose the correct answer for the following objectives :

10. If A and B are 3 sets, then $A \cap (B \cup C)$ is equal to :

(a) $(A \cup B) \cap (A \cup C)$ (b) $(A \cap B) \cap (A \cap C)$

(c) $(A \cap B) \cup (A \cap C)$ (d) None of these 1

11. $\sin (180° + \theta) \sin (180° - \theta) \operatorname{cosec}^2 \theta$ is equal to :

(a) 1 (b) – 1

(c) 0 (d) None of these 1

12. The 4th term of a G.P is 2, then the product of first 7 terms is :

(a) 64

(b) 128

(c) 32

(d) $\frac{1}{27}$ 1

13. If the third term of an AP is 12 and the seventh term is 24, then the 10th term is :

(a) 36

(b) 30

(c) 39

(d) 33 1

14. The chance that an event occur or does not occur is :

(a) 0

(b) 1

(c) 2

(d) None of these 1