B–7–Z

Roll No. ....

Total No. of Questions : 31] [Total No. of Printed Pages : 8

12^{th}ARM(SZ)JKUT2024
1107–Z
MATHEMATICS

Time : 3 Hours] [Maximum Marks : 80

SECTION–A

(OBJECTIVE TYPE QUESTIONS/ MULTIPLE CHOICE QUESTIONS) 1 each

1. In determinant

$$\begin{vmatrix} -1 & 2 \\ -6 & 7 \end{vmatrix}$$

cofactor of 2 is :

(A) 6

(B) –6

(C) –1

(D) 7

2. If A and B are two matrices, then (AB)′ = ...

3. Order of differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$$

is :

(A) 1

(B) 2

(C) 4

(D) Not defined

4. Number of points of discontinuity of the function

$$f(x) = \frac{1}{x^2 - 5x + 6}, x \neq 2, 3$$

is :

(A) 1

(B) 2

(C) 3

(D) 0

5. $\int \sec x \, dx$ is equal to $\log(\sec x + \tan x) + c$. (True/False)

6. The function $f(x) = 2x^2 - 3x$ is decreasing for ....

7. Derivative of $\cos(\sqrt{x})$ is :

(A) $\frac{-\sin(\sqrt{x})}{2\sqrt{x}}$

(B) $\frac{\sin(\sqrt{x})}{2\sqrt{x}}$

(C) $2\sqrt{x}\cos(\sqrt{x})$

(D) $2\sqrt{x}\sin(\sqrt{x})$

8. Direction cosines of z-axis are :

(A) 1, 0, 0

(B) 0, 1, 0

(C) 0, 0, 1

(D) 0, 0, 0

9. $\hat{i}.(\hat{j} \times \hat{k})$ is equal to :

(A) 1

(B) −1

(C) 0

(D) 2

10. Define Linear programming problem.

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Check the injectivity and subjectivity of the function $f : \mathbf{N} \to \mathbf{N}$ given by $f(x) = x^2$.

12. Find the principal value of

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right).$$

13. Prove that the function $f(x) = \log \sin x$ is strictly increasing on

$$\left(0, \frac{\pi}{2}\right).$$

14. Evaluate the product : https://www.jkboseonline.com

$$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b}).$$

15. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

16. Find $\int \sin x \sin(\cos x)\, dx$.

17. Evaluate :

$$\int_0^{\pi/4} \sin 2x \, dx.$$

18. If

$$P(A) = \frac{3}{5}$$

and

$$P(B) = \frac{1}{5}$$

find $P(A \cap B)$ if A and B are independent events.

19. Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

20. Show that the matrix :

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

is a symmetric matrix.

SECTION–C

(SHORT ANSWER TYPE QUESTIONS) 4 each

21. Find the general solution of the differential equation :

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x.$$

22. Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx.$$

23. If

$$x = \sqrt{a^{\sin^{-1} t}},\ y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}.$$

24. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda\,(\hat{i} - \hat{j} + \hat{k})$$

and

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu\,(2\hat{i} + \hat{j} + 2\hat{k}).$$

25. If

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

and

$$\vec{c} = 3\hat{i} + \hat{j}$$

are such that :

$$\vec{a} + \lambda\vec{b}$$

is perpendicular to $\vec{c}$, then find the value of λ.

26. Solve the linear programming problem graphically

Minimize $z = -3x + 4y$

subject to :

$$x + 2y \leq 8,$$

$$3x + 2y \leq 12,$$

$$x \geq 0,\ y \geq 0.$$

27. Show that the relation R in the set Z of integers given by R = $\{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.

28. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SECTION–D

(LONG ANSWER TYPE QUESTIONS) 6 each

29. If :

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}.$$

verify A (adj A) = (adj A)A = |A| I.

Or

Let :

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

verify $(AB)^{-1} = B^{-1}.A^{-1}$.

30. **Evaluate :**

$$\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\, dx.$$

Or

Evaluate :

$$\int \frac{(3x - 1)\, dx}{(x - 1)(x - 2)(x - 3)}.$$

31. If $y = Ae^{mx} + Be^{nx}$, show that :

$$\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0.$$

Or

Differentiate w.r.t. x :

$$(\log x)^x + x^{\log x}.$$

B–7–Y

Roll No.

Total No. of Questions : 30 | [Total No. of Printed Pages : 8

12th ARM(SZ)JKUT2024
1107-Y
MATHEMATICS

Time : 3 Hours] [Maximum Marks : 80

SECTION-A

(OBJECTIVE TYPE QUESTIONS/
MULTIPLE CHOICE QUESTIONS) 1 each

1. In determinant

$$\begin{vmatrix} -1 & 3 \\ 6 & 10 \end{vmatrix}$$

cofactor of 10 is

(A) 1

(B) –1

(C) 3

(D) 6

2. If matrix B is inverse of matrix A, then AB = BA = ...

Turn Over

3. Order of differential equation

$$x^2 \frac{dy}{dx} = -y^5$$

is

(A) 1

(B) 2

(C) 5

(D) 0

4. Derivative of $\cos (xe^x)$ is :

(A) $-e^x (x + 1) \sin (xe^x)$

(B) $e^x (x + 1) \sin (xe^x)$

(C) $-\sin (xe^x)$

(D) $\sin (xe^x)$

5. Number of points of discontinuity of the function $f(x) = x + 5$ is :

(A) 1

(B) 2

(C) 3

(D) 0

6. $\int \tan x \, dx$ is equal to $\log \sec x + c$. (True/False)

7. The function $f(x) = x^2 + 2x + 3$ is decreasing for ....

8. Direction cosines of y-axis are :

(A) 1, 0, 0

(B) 0, 1, 0

(C) 0, 0, 1

(D) 0, 0, 0

9. Unit vector along the direction of the vector $\hat{i} + \hat{j} + \hat{k}$ is :

(A) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

(B) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{2}}$

(C) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{3}$

(D) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{2}$

10. Define feasible region.

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Check the injectivity and subjectivity of the function $f : R \to R$ given by $f(x) = x^2$.

12. Find the principal value of

$$\tan^{-1}(-\sqrt{3}).$$

13. Prove that the function $f(x) = \log \cos x$ is decreasing for

$$\left(0, \frac{\pi}{2}\right).$$

14. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$.

15. Show that :

$$(\vec{a} - \vec{b}) \times (a + \vec{b}) = 2(\vec{a} \times \vec{b}).$$

16. Find :

$$\int \frac{1}{x + x \log x}\, dx.$$

17. Evaluate :

$$\int_0^{\pi/2} \cos 2x \, dx.$$

8. If

$$P(A) = \frac{3}{5}$$

and

$$P(B) = \frac{1}{5}$$

find $P(A \cap B)$ if A and B are independent events.

9. Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$

10. Show that the matrix

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

is a symmetric matrix.

SECTION–C

(SHORT ANSWER TYPE QUESTIONS) 4 each

1. Find the general solution of the differential equation :

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x.$$

2. Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx.$$

23. If

$$x = \sqrt{a^{\sin^{-1} t}},\ y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}.$$

24. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} - \hat{k})$$

and

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

25. If

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

and

$$\vec{c} = 3\hat{i} + \hat{j}$$

are such that :

$$\vec{a} + \lambda\vec{b}$$

is perpendicular to $\vec{c}$, then find the value of λ.

26. Solve the linear programming problem graphically :

Minimize $z = -3x + 4y$

subject to :

$$x + 2y \le 8,$$

$$3x + 2y \le 12,$$

$$x \ge 0,\ y \ge 0.$$

27. Show that the relation R in the set Z of integers given by R = {(*a*, *b*) : 2 divides *a* – *b*} is an equivalence relation.

28. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SECTION–D

(LONG ANSWER TYPE QUESTIONS) 6 each

29. If :

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

verify A (adj A) = (adj A)A = |A| I.

Or

Let

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

verify $(AB)^{-1} = B^{-1}.A^{-1}$.

30. Evaluate :

$$\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\, dx.$$

Or

Evaluate :

$$\int \frac{(3x - 1)\, dx}{(x - 1)(x - 2)(x - 3)}.$$

31. If $y = Ae^{mx} + Be^{nx}$, show that :

$$\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0.$$

Or

Differentiate w.r.t. x :

$$(\log x)^x + x^{\log x}.$$

B–7–X

Roll No. ....

Total No. of Questions : 31]

[Total No. of Printed Pages : 8

12th ARM(SZ)JKUT2024

1107–X

MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 80

SECTION–A

(OBJECTIVE TYPE QUESTIONS/ MULTIPLE CHOICE QUESTIONS)

1 each

1. In determinant

$$\begin{vmatrix} 3 & 6 \\ -2 & 5 \end{vmatrix}$$

cofactor of –2 is :

(A) 6

(B) 3

(C) 5

(D) –6

2. For a square matrix A. A(adj A) = (adj A) A = ...

Turn Over

3. Order of differential equation :

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$$

is :

(A) 3

(B) 2

(C) 1

(D) 4

4. Derivative of $e^{\log \sin x}$ is :

(A) $\sin x$

(B) $\cos x$

(C) $\sec x$

(D) $\tan x$

5. Number of points of discontinuity of the function :

$$f(x)=\frac{1}{x-5}, x\neq 5$$

is :

(A) 1

(B) 2

(C) 3

(D) 5

6. $\int xe^{x^2}\,dx$ is equal to $\frac{e^{x^2}}{x} + c$. (True/False)

7. The function $f(x) = 3x + 5$ is increasing for ....

8. Direction cosines of x-axis are :

(A) 1, 0, 0

(B) 0, 1, 0

(C) 0, 0, 1

(D) 0, 0, 0

9. Magnitude of Vector $\hat{i} + 2\hat{j} + 3\hat{k}$ is :

(A) $\frac{1}{\sqrt{14}}$

(B) $\sqrt{14}$

(C) 1

(D) 10

10. Define objective function.

SECTION–B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Check the injectivity and subjectivity of the function $f : Z \rightarrow Z$ given by $f(x) = x^2$.

12. Find the principal value of

$$\sin^{-1}\left(-\frac{1}{2}\right).$$

13. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

14. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

15. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

16. Find :

$$\int \frac{(\log x)^2}{x}\,dx.$$

17. Evaluate :

$$\int_0^1 \frac{dx}{\sqrt{1 - x^2}}.$$

18. If

$$P(A) = \frac{3}{5}$$

and

$$P(B) = \frac{1}{5}$$

find $P(A \cap B)$ if A and B are independent events.

19. Evaluate $P(A \cup B)$ if $2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

20. Show that the matrix :

$$A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

is a symmetric matrix.

SECTION–C

(SHORT ANSWER TYPE QUESTIONS) 4 each

21. Find the general solution of the differential equation :

$$x \log x \frac{dy}{dx} + y = \frac{2}{x} \log x.$$

22. Evaluate :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx.$$

23. If

$$x = \sqrt{a^{\sin^{-1} t}},\ y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}.$$

24. Find the shortest distance between the lines :

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

and

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

25. If

$$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k},\ \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$$

and

$$\vec{c} = 3\hat{i} + \hat{j}$$

are such that :

$$\vec{a} + \lambda\vec{b}$$

is perpendicular to $\vec{c}$, then find the value of λ.

26. Solve the linear programming problem graphically :

Minimize $z = -3x + 4y$

subject to :

$$x + 2y \leq 8,$$

$$3x + 2y \leq 12,$$

$$x \geq 0,\ y \geq 0.$$

27. Show that the relation R in the set Z of integers given by R = $\{(a, b) : 2 \text{ divides } a - b\}$ is an equivalence relation.

28. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

SECTION–D

(LONG ANSWER TYPE QUESTIONS) 6 each

29. If :

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

verify A (adj A) = (adj A)A = |A| I.

Or

Let :

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

verify $(AB)^{-1} = B^{-1}.A^{-1}$.

30. Evaluate :

$$\int_{-1}^{1} 5x^4 \sqrt{x^5 + 1}\, dx.$$

Or

Evaluate :

$$\int \frac{(3x-1)\,dx}{(x-1)(x-2)(x-3)}.$$

31. If $y = Ae^{mx} + Be^{nx}$, show that :

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0.$$

Or

Differentiate w.r.t. x :

$$(\log x)^x + x^{\log x}$$